Research Annals of Industrial and Systems Engineering

www.raise.reapress.com

Res. Ann. Ind. Syst. Eng. Vol. 1, No. 4 (2024) 244–261.

Paper Type: Original Article

Multi-Objective Uncertain Mathematical Programming for Urban Water Supply System

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Citation:

Received: 15 June 2024 Revised: 21 August 2024 Accepted: 27 November 2024	Daneshmand-Mehr, M., & Abolghasemian, M. (2024). Multi-objective uncertain mathematical programming for urban water supply system. *Research annals of industrial and systems engineering, 1*(4), 244-261.

Abstract

Water supply management is essential in social development, ecosystem sustainability, and environmental management. For this purpose, this study presents a planning framework for city water supply management based on water security indicators in Rasht city, Guilan province, to formulate and implement a water security policy using mathematical modeling. In this article, a dual-purpose model is used to manage water supply, minimizing costs, water shortages, and waste. Also, in the proposed model, water consumption in the urban area is treated as uncertain using a fuzzy approach. According to the results, evaluating the accuracy and validity of the model for the water supply system from different water resources shows that increasing treatment capacity and water resources in Guilan province can reduce treatment costs and shortages, and reduce wastewater. Regarding the parameters that positively affect the amount of input water, it is necessary to implement appropriate systems to control input water and manage these valuable resources. Eventually, forecasting the amount of shortage in the studied area over the next 100 years indicates a linear trend: the shortage increases in each period due to increased population and decreased precipitation.

Keywords: Water resource management, Optimization, Wastewater management, Water supply, System dynamics.

1 | Introduction

Water resources are of great importance for socioeconomic development, food production, and environmental conservation. Thus, its exploitation is a vital matter for the development of humanity, Ghannem et al. [1].

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https://doi.org/10.22105/raise.vi.74

Water is considered a vital substance for human life and social development. The problem of water resource management has worsened due to rapid economic development. The shortage of water resources significantly limits social and economic development and threatens drinking water security. For this reason, the water shortage crisis is increasing due to climate change and human activities. In addition, demand for water resources has gradually increased with the development of industrial and agricultural production and the social economy [2].

Furthermore, population growth, urbanization, climate change, mismanagement of water resources and water distribution systems, and inappropriate programming increase water stress in many countries worldwide. Although optimization of water distribution systems and integrated water management techniques have been conducted in many cities, no integrated study has been found regarding urban water systems, programming, and management [3]. Due to population growth, limited water resources, and other pressures on urban water systems, urban water management has become a primary concern for urban policymakers in recent years, as they seek to develop efficient management solutions [4]. Thus, solving network design problems has become an interesting topic in the field of wastewater management and water security because of increased population, resource shortage, environmental concerns, and the increasing need for sustainable solutions for future policy plans, especially in the areas where the demand for wastewater treatment has increased significantly due to high immigration, rapid industrialization, and tourism activities to make this topic more critical and dynamic Demirel et al. [5]. Following the long-term effects of urban water services is a background for the future and a critical scientific foundation for the sustainable development of systems Shiu et al. [6] since the appropriate exploitation of water resources is considered as an effective measure to realize the optimal allocation of water resources and can effectively decrease the regional water resource shortages, flood disasters, and other social problems, and play a key role in supporting the strategic development of water resources. Water supply for urban supply systems is a global effort and a big challenge for water resource managers in large urban areas. Water supply requires integrated management of water quantity and quality, and adaptation of the system to land use and climate change in the region [7].

Current water security models assess a system's security status using indicators of quantity, quality, and compliance. Moreover, existing studies address this topic using mathematical optimization-based approaches that require substantial computational effort. In addition to mathematical optimization studies, an alternative approach based on the system dynamics method is proposed in this study to evaluate the complex dynamic and nonlinear structure of water and wastewater network design problems and water supply since the developed framework can provide an evaluation method for the water life cycle in the long term in a dynamic way to evaluate the temporal changes and the effects to deal with the challenges related to water security. The proposed SD simulation model has been designed for a populated tourist spot in Guilan province, located in the north of Iran and the southwest of the Caspian Sea. Accordingly, the most significant innovation and unique contribution of this study are as follows:

I. Establishing a three-phase water supply system, including the processes of water production, distribution, and purification

II. Creating a quantitative relationship in the urban water supply system and evaluating system performance based on it.

III. Providing integrated programming to determine the optimal status of water supply protocols and select the most appropriate water supply strategy based on various programming periods for policy makers.

IV. Establishing an optimal programming cycle for a real-world water supply system by combining the bi-objective mathematical model and system dynamics modeling methods.

V. Considering an uncertain value for the amount of water consumed in the time period to make the modeling closer to reality.

2 | Literature review

Golpira and Tirkalaei [8] evaluated the social and economic effects at different levels of environmental flow allocation in the Wei River basin in China using the systems dynamics model and VENSIM software. In this report, four methods were presented for social and economic growth, and four methods for environmental water allocation. The results indicated that the developed system dynamics model captures the system's dynamic behavior well in the studied area. Nepal and Teren [9] addressed the use of the system dynamics method in simulating the cultivation pattern of the irrigation and drainage network on the right side of the Isfahan waterfall. The results revealed that maintaining or not maintaining the limit on the total area under cultivation in the base year yields the highest income relative to the cost. In addition, they used the VENSIM system dynamics model to simulate electricity production from the Karun 1, Karun 3, and Karun 4 hydroelectric reservoirs. The design steps involved defining the required decision variables, equations, and formulas to measure the energy production of the reservoirs. The results indicated that energy production in the Khorasan 1 reservoir, including the Karun 4 and 3 reservoirs, can increase by an average of 20% without significant changes to the evaluation criteria. Song et al. [10] evaluated different scenarios for water resource allocation of Choghakhor dam using the system dynamics method. The VENSIM dynamic model was designed and implemented to utilize the water resources of the Choghakhor dam and meet downstream needs. In addition, the input amounts to the dam reservoir and evaporation from the dam reservoir were predicted using a SARIMA time-series model for the next 5 years. The results showed that the Choghakhor dam can supply the water needs of 1600 hectares of water-consuming lands in the most optimal state. In this regard, the amount of supply for agricultural and environmental needs is 83 and 95, respectively. Tian et al. [11] used the PSO, genetic, and ant colony optimization algorithms to study wastewater network optimization. They concluded that the PSO algorithm can reach the optimal solution faster with fewer iterations. Zhu et al. [12] developed domestic and industrial water demand models based on a bi-objective model to balance economic and environmental costs in an urban water supply system across three phases: water production, distribution, and treatment. Then, a system dynamics-based model is applied to test the performance of the water supply system under short-, medium-, and long-term programming scenarios. Yu et al. [13] studied stakeholder competition in the operation of a multi-purpose ecological reservoir to satisfy economic, social, and ecological needs. For this purpose, a multi-objective game-theoretic model was developed to determine water discharge over 10 days to satisfy the triple needs of water —electricity production, socioeconomic consumption, and the environment. Dimello et al. [7] proposed a new method for evaluating water security based on the ranking of pressure indicators, which apply to a drainage basin such as water demand, regular and random pollutants, drought, and environmental changes (e.g., the share of forest vegetation) according to risk assessment in terms of pressure characteristics such as severity, occurrence, and detectability. Dang et al. [14] formulated a multi-objective water resource allocation model to optimize efficiency and equity with sustainability (ecological river flow) as a constraint. For this purpose, the NSGA-II algorithm was used to extract Pareto solutions in a water resource allocation system. The results showed that Pareto solutions can show the opposite relationship between efficiency and equity in water resource allocation. Demirel et al. [5] proposed an alternative approach based on the system dynamics method to select a location for wastewater treatment facilities and to design wastewater treatment networks. The proposed SD simulation model was designed for an area in Antalya, Turkey. This model can determine the location and timing of constructing a new wastewater treatment plant and a public wastewater network for five areas in downtown, based on cost considerations during 2015-2040. Guilani et al. [15] provided a sustainable selection method for the location of water treatment facilities using the best-worst method. Moreover, this model selects appropriate technologies for the treatment plant, manages water leakage across the entire transmission network through renovation, and selects different transmission technologies. Based on the obtained results, the interaction of water and energy has received special attention in this network. Shiu et al. [6] proposed a method for dynamic life cycle assessment that considers temporal changes and the challenges of water treatment facilities, based on life cycle assessment principles applied to system dynamics models. This model was then applied to a water treatment plant in the Kinmen Islands, Taiwan. The SD model can simulate long-term growth in water demand across the domestic, agricultural, livestock, and manufacturing sectors. Zhu [16] developed an integrated framework for climate change and analyzed its effects on social-environmental aspects across many areas, including wastewater treatment, energy transmission, waste management, land management, and ocean management. Heydari-Kushalshah et al. [17] presented a mathematical model for water supply management. The proposed model can minimize water loss and shortages in an urban area.

Based on the above literature review, the literature fails to present an appropriate optimization model for the effective management of urban water supply resources, given the area's conditions. Thus, introducing a water supply system that includes production, distribution, and treatment processes is recommended to establish a quantitative relationship within the urban water supply system and to describe system performance based on this relationship. Regarding the most significant advantage of this study, it has been conducted to strengthen water resources management through linear multi-objective mathematical modeling that accounts for factors affecting water supply in a high-rainfall province such as Guilan, Iran. Based on the presented modeling, total efficiency and economic benefits are achieved across the domestic, industrial, and agricultural sectors. As a result, the most significant research gaps are as follows:

I. A framework that tracks the optimal response to water supply protocols by changing the programming period according to an integrated method, and determines the most appropriate water supply strategy and programming period for policy makers.

II. Establishing a three-phase water supply system, which involves the processes of water production, distribution, and treatment.

III. Defining a quantitative relationship in the urban water supply system to measure the performance of the system according to this relationship.

IV. Defining an optimal programming period for a real-world water supply system by combining the bi-objective programming and system dynamics methods.

V. Presenting a multi-objective uncertain mathematical programming for the urban water supply system.

3 | Method

This study focuses on the use of mathematical modeling and system dynamics. In this regard, a bi-objective mathematical model is presented to optimize water resources. This model can simultaneously minimize overall system and wastewater costs, which are primary challenges in water resource management. Also, in the proposed model, water consumption in the urban area is treated as uncertain using a fuzzy approach. Numerous efforts have been made to manage wastewater, but no satisfactory results have been achieved. This study, which focuses on Guilan province, a rainy province in northern Iran, presents a mathematical model that minimizes costs and wastewater use while returning water resources to the urban and rural water supply cycle. Guilan province is considered one of the few regions in Iran with abundant water. This province can serve as a model for greater productivity in water resources for other provinces in Iran.

3.1 | Urban Water Supply System Structure

The water resources programming and management structure in the urban water supply system can be regarded as a multi-objective optimization problem due to multiple objectives, constraints, and highly diverse variables. For this purpose, *Fig. 1* shows the general structure of the urban water supply system in the studied sector.

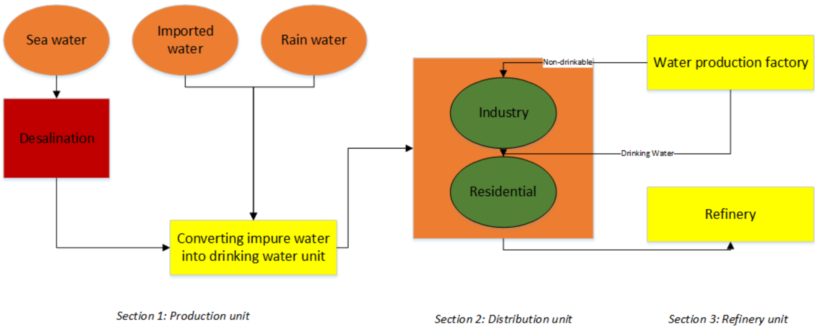

Fig. 1. The general model of the studied urban water supply system.

As shown in *Fig. 1,* three main sectors are involved in the intended urban water supply system. The first sector, the production sector, includes various water resources such as seawater, rainwater, imported water from neighboring countries, underground water reservoirs, and others. In addition to these water resources, the desalination unit operates in the production sector alongside other water sources to convert Caspian Sea water into drinking water. Further, the treatment unit there converts water from reservoirs, rain, imported water, and wastewater into drinking water. The second sector is the distribution network, or the water consumption sector, which serves two major groups of water resource subscribers: industrial and residential. Finally, the water used in the distribution sector is collected and transferred to the recovery (reproduction) and treatment sectors, so that a portion of it is returned to the consumption cycle (industrial and residential consumption), if possible. In this sector, there are water treatment plants and drinking water production plants. Part of the water entering the water treatment plants is poured into the sea without any special treatment. The other part is transferred to water production plants after treatment to provide water for the industrial and residential sectors. An inclusive urban water supply system for all types of residential and industrial consumption has a complex mechanism that requires a comprehensive, optimal management plan to achieve its critical goals. Since there is an optimization problem in the programming and management of urban water systems, a modeling optimization problem should be formulated within a standard structure to address it. The following section presents the mathematical model for optimal management and programming of the urban water system.

3.2 | Mathematical Modeling of the Problem

This section explains the research assumptions, parameters, variables, and the objective functions and their constraints to introduce the mathematical model.

Assumption

The most significant hypotheses of this study are as follows:

I. The parameters are determined definitively.

II. The model is multi-period.

III. Every area includes the city and its subordinate villages.

IV. Water resources are considered constructed.

V. The construction of treatment systems is considered a variable.

VI. In addition to the construction cost, the treatment systems have a management fee per kilometer of water resources.

VII. Water supply resources have fees.

VIII. Energy consumption and its cost are considered for water resources and treatment systems.

IX. Wastewater and shortage are considered in the present model.

Symbolization

Table 1 presents all the indices, parameters, and variables considered in the mathematical model.

Table 1. Mathematical model symbolization.

Indices		
Symbol	**Description**	
I	Number of areas	
J	Number of treatment systems	
K	Number of water supply resources	
T	Time	
Parameters		**Type of Parameters**
DEM_{it}	The amount of water consumption in area i in time t	Uncertain
FCJ_{ij}	Cost of water treatment system construction j in area i	Deterministic
FCK_{ik}	Cost of water supply resource k in area i	
VCJ_j	Cost of settling the water treatment system j	
EW_{it}	The amount of water resources entering the area i in time t	
$CAPJ_{ij}$	Capacity of water treatment system j in area i	
$CAPK_{ik}$	Water resource capacity k in area i	
ENC_j	The amount of energy consumption for water treatment in the water treatment system j	
$UENC_j$	Cost of energy consumption per unit for water treatment in the water treatment system j	
$ENCK_k$	The amount of energy consumption in the water supply resource k	
$UENCK_k$	Cost of energy consumption per unit for the water supply resource k	
TC_{kji}	Cost of water transmission from water supply resource k to water treatment system j in area i	
Variables		**Type of Variables**
X_{ij}	Its value is 1 water treatment system j is constructed in area i; otherwise, it is zero.	Binary
U_{kji}	The amount of water transmission from water supply resource k to water treatment system j in area i	Continuous
Y_{ik}	The amount of water resources available in water supply resource k in area i	
Z_{jt}	The amount of treated water in the water treatment system j at time t	
WW_{it}	The amount of wastewater in area i in time t	
LW_{it}	The amount of water shortage in area i in time t	

Objective functions

The first objective function of the raised problem in *Eq. (1)* is to minimize the cost of water supply, including the energy cost, amount of energy consumption, and cost of constructing a treatment system, cost of managing water resources, the cost of managing the treatment system, and the cost of transmitting water resources. In addition, the second objective of the problem in *Eq. (2)* is to minimize wastewater and water shortage for the entire system, where wastewater is measured by demand and consumption, and the input determines the shortage.

$$\begin{aligned}\min z1 = &\sum_{i}^{I}\sum_{j}^{J} FCJ_{ij} \,.\, X_{ij} + \sum_{i}^{I}\sum_{k}^{K} FCK_{ik} \,.\, Y_{ik} + \sum_{j}^{J}\sum_{t}^{T} VCJ_j \,.\, Z_{jt} + \sum_{j}^{J}\sum_{t}^{T} ENC_j \,.\, UENC_j . Z_{jt} \\ &+ \sum_{i}^{I}\sum_{k}^{K} ENCK_k \,.\, UENCK_k . Y_{ik} + \sum_{i}^{I}\sum_{j}^{J}\sum_{k}^{K} TC_{kji} \,.\, U_{kji},\end{aligned} \tag{1}$$

$$\min z2 = \sum_{i}^{I}\sum_{t}^{T} WW_{it} + LW_{it}. \tag{2}$$

Constraints

$$\sum_{i=1}^{I} X_{ij} = 1, \quad (3)$$

$$U_{kji} \leq X_{ij}, \quad (4)$$

$$\sum_{i=1}^{I} U_{kji} = 1, \quad (5)$$

$$Y_{ik} \leq CAPK_{ik}, \quad (6)$$

$$Z_{jt} \leq CAPJ_{ij}, \quad (7)$$

$$Z_{jt} \leq M.X_{ij}. \quad (8)$$

$$WW_{it} = EW_{it} - DEM_{it}, \quad (9)$$

$$LW_{it} = DEM_{it} - \sum_{k=1}^{K} Y_{ik}, \quad (10)$$

$$\sum_{j=1}^{J} Z_{jt} \leq \sum_{k=1}^{K} Y_{ik}, \quad (11)$$

$$X_{ij} \in \{0,1\}, \quad (12)$$

$$U_{kji} \in \{0,1\}, \quad (13)$$

$$Y_{ik} \geq 0, \quad (14)$$

$$Z_{jt} \geq 0, \quad (15)$$

$$WW_{it} \geq 0, \quad (16)$$

$$LW_{it} \geq 0. \quad (17)$$

Based on *Eq. (3)*, every treatment center can be constructed in a single area. *Eq. (4)* shows that the construction of those centers determines the transmission of resources from treatment centers. According to *Eq. (5)*, water transmission from water resources to treatment centers is only significant in one area. In addition, *Eq. (6)* reveals the capacity limitation of water resources. *Eq. (7)* shows the limitation of the treatment center capacity. *Eq. (8)* indicates that if a treatment plant is present, it can include water resources. Furthermore, *Eq. (9)* addresses wastewater calculation in the system, and *Eq. (10)* considers shortage calculation in the system. *Eq. (11)* indicates that water resources in treatment centers cannot be more than the water resources in water resource systems. *Eqs. (12) – (17)* present the type of variables in the problem, which are binary integer and continuous (positive).

In this model, the amount of water consumption in the urban area (demand amount) is considered uncertain. Therefore, to make the model more closely aligned with the real world, it is formulated in a fuzzy form. To address data uncertainty, we intend to treat the demand parameter as realistically as possible while accounting for uncertainty. This hypothesis has been ignored in the basic model presented by Heydari-Kushalshah et al. [18]. In fact, we want to provide a plan to cover the unknown demand in the form of a chain. In this research, we assume that in a fixed period of time, the amount of water consumption in area i in time t (DEM_{it}) is assumed equal to S_t. We assume that demand in period t is continuously uncertain and varies with actual conditions. Uncertainty in the water supply chain can arise for any reason, including decreased rainfall, the onset of the hot season, etc. This parameter is considered a non-deterministic distance$[\widetilde{S_t} - \widehat{S_t}, \widetilde{S_t} + \widehat{S_t}]$. According to the uncertain space, like S_k, the bounded symmetric form $\widetilde{S_t}$ is $\widehat{S_t} = \rho\widetilde{S_t}$ where $\widetilde{S_t}$ the estimated value for the demand parameter is. In fact, $\widetilde{S_t}$ is the fluctuation rate of the demand parameter, and $\rho > 0$ is the level of uncertainty. Therefore, the goal is to estimate a set of random demands to provide the most supply at the lowest possible cost. The main decisions in this case are estimating demand and determining its starting time. Therefore, we consider a binary variable $Z_i$and variable $y_{it}$for each demand. For $1 \leq c \leq C$, the value of Z_i will equal 1 if the demand is met. Otherwise, it will be zero. For $1 \leq t \leq T$ and$1 \leq i \leq I$, the value of $y_{it}$is equal to one if the demand for area i starts in period t. Otherwise, its value will be zero. Since each demand can only occur once in the planning horizon H, then we have:

$$Z_i = \sum_{t=1}^{T} y_{it}, \quad 1 \leq i \leq I;\ 1 \leq t \leq T;\ \ i = 1, \dots, I;\ t = 1, \dots, T. \tag{18}$$

Eq. (18) allows us to transform the variable $Z_i$like a continuous variable. Because the independent variables in Eq. (18) are either 0 or 1*, their sum is nonnegative*. Now, if a demand is satisfied in period k, we can calculate y_{ik}via the continuous variable (19).

$$y_{it} = \sum_{t'=\max\{1,t+1\}}^{T} y_{it}, \quad 1 \leq i \leq I;\ \ 1 \leq t \leq T;\ i = 1, \dots, I;\ t = 1, \dots, T. \tag{19}$$

Therefore, to account for the uncertain and unknown demand, the above equations are added to the proposed model.

3.3 | Problem Solving Method: Epsilon Constraint

The solution method in this study includes the exact method, which uses the epsilon-constraint algorithm. This section explains the algorithm, and the problem is solved using the epsilon-constraint method. In this regard, the first objective is considered the main objective, and the second objective is constrained to the upper limit epsilon and applied to the problem constraints. Based on the epsilon method, the multi-objective model's constraint is reduced to a single-objective model 18. In this method, one objective function is selected, and the other objective functions are converted into constraints based on the amounts determined by the decision maker or modeler. In addition, the problem is formulated as a single-objective linear programming model and solved using the linear programming method. The main advantage of the epsilon constraint is its applicability to non-convex solution spaces, since methods such as weighted objective combination lose efficiency in such spaces. The computational time of an algorithm is a significant feature for evaluating it. The use of the meta-heuristic algorithm leads to a sharp reduction in computational time, since one of the major weaknesses of algorithms based on exact search, including the epsilon method, is their high computational cost. A framework presented by Pirouz and Khoram [19] is one of the modified versions of the epsilon constraint method. Abolghasemian et al. [20–22], have recently recommended this version due to its two significant advantages. The smaller search space for finding non-dominant points is one of this method's advantages. Another advantage of this method is its shorter execution time than the original method. Based on this method, the single-objective optimization problem is solved for each objective, and then the step length is determined. Then, a set of appropriate points is generated. Finally, the single-objective optimization is solved and the Pareto frontier is estimated.

$$\begin{aligned} &\min f_1(x), \\ &f_i(x) \leq e_i, \\ &x \in X. \end{aligned} \tag{20}$$

The first objective in *Eq. (18)* is considered the main objective, and the second to $n-th$ objectives are limited to the maximum value e_i. In the epsilon constraint method, different solutions are obtained, which may not be effective by changing the values of e_i. The problem can be solved by modifying the aforementioned model, known as the modified epsilon-constraint method. In this method, the previous equation is rewritten as *Eq. (21)*.

$$\min f_1(x) - \sum_{i=1}^{2} \phi_i s_i,$$

$$fi(x) + s_i = e_i, \tag{21}$$

$$x \in X,$$

$$s_i \geq 0.$$

Where s_i represents the auxiliary nonnegative variables and ϕ_i shows a parameter for the normalization of objectives.

3.4 | Development of System Dynamics Model

The simulation using the system dynamics approach in this study involves the following steps:

Development of the simulation model

All key effective variables are specified in the cause-effect loop diagram and then converted into a flow diagram using VENSIM software to analyze the effective factors. Then, an integrated SD model is simulated based on the relationships between the variables.

Definition of cause-and-effect relationship

A cause-and-effect diagram is often used to indicate the dynamic interactions between system elements, with the positive and negative effects of each element on the others shown with (+) and (-) signs, respectively.

Drawing the flow diagram

The flow diagram is quantitative and depends on the definitions of certain variables for problem-solving.

The variables used in the system dynamics in this study are those of the mathematical model, whose values are obtained by solving the model and entered as parameters in VENSIM. *Table 2* presents the cause-and-effect variables required for the system dynamics model.

Table 2. System dynamics model variables.

Dynamics Model Parameters	
Description	**Symbol**
The amount of water resources available in the water supply source k in area i	Y_{ik}
The amount of treated water in the water treatment system j in time t	Z_{jt}
The amount of wastewater in area i in time t	WW_{it}
The amount of water shortage in area i in time t	LW_{it}
Dynamic Model Variables	
Total cost of the water resources system	F1
Total amount of shortage and wastewater in the system	F2

4 | Results

4.1 | Case Study

Urban water and wastewater companies were established in Iran with the approval of the Islamic Council on January 1, 1991. These companies aim to create and operate urban water facilities and collect, transfer, and treat wastewater within the legal limits of the cities in each province. The regional water companies already fulfilled the relevant tasks in part of the province and by municipalities in smaller cities. After the establishment of the urban water and wastewater company in Iran, these functions were transferred to the new company. The same tasks in the rural sector were assigned to the newly established rural water and wastewater companies on December 13, 1995. Before that, such services were conducted at the Jihad, construction, and health centers in the villages. By merging the urban and rural water and wastewater companies in March 2019, all service functions in cities and villages for the supply and distribution of sanitary and drinking water, as well as the collection, transmission, and treatment of wastewater, have been entrusted to the integrated water and wastewater companies. In general, water and wastewater companies are non-governmental, financially and legally independent, and governed by commercial law.

4.2 | Results of Numerical Solution for the Model

This section presents the results of solving the model and its validation across different dimensions (small, medium, and large). For this purpose, *Table 3* presents the problem's dimensions.

Table 3. Dimensions of the problem and predetermined parameters.

Problem Dimensions	Problem	Area I	Treatment System J	Water Supply Resourcesk	Time T
Small	1	15	12	20	10
	2	16	12	20	10
	3	17	12	20	10
	4	18	12	20	10
	5	19	12	20	10
	6	20	12	20	10
	7	21	12	20	10
Medium	8	22	14	25	12
	9	23	14	25	12
	10	24	14	25	12
	11	25	14	25	12
	12	26	14	25	12
	13	27	14	25	12
Large	14	28	16	30	15
	15	29	16	30	15
	16	30	16	30	15
	17	31	16	30	15
	18	32	16	30	15
	19	33	16	30	15
	20	34	16	30	15

As observed, the problem dimensions change in each example above. An increase in dimensions affects the value of the objective functions and the calculation time, provided the modeling is conducted correctly. For this purpose, *Table 4* presents the objective function values and the calculated time.

Table 4. Results of the mathematical model solution.

Problem Dimensions	Problem	Supply Cost (Z_1)	Wastewater And Shortage (Z_2)	Calculation Time
Small	1	142744	3309361	18
	2	144184	3324486	36
	3	145869	3337927	48
	4	147512	3352119	64
	5	148587	3366940	76
	6	149944	3383915	89
	7	151091	3396378	107
Medium	8	153059	3414299	120
	9	154107	3424478	139
	10	155121	3442370	149
	11	156516	3459297	167
	12	158157	3471627	177
	13	160079	3486565	189
Large	14	161459	3497552	204
	15	162997	3514357	223
	16	164203	3524721	236
	17	165461	3540257	251
	18	166838	3558616	268
	19	168390	3578511	281
	20	170349	3590094	295

Table 4 presents the results from 20 examples, including the cost of water supply and the amount of wastewater. To check the model's accuracy, the results from the diagrams of these amounts are presented. For example, as shown in *Fig. 2,* the cost of the entire system increases with the problem's dimensions, indicating the model's accuracy and validity. *Fig. 3* shows the amounts of wastewater and the shortage. As a result, an increase in dimensions leads to a natural increase in wastewater and a shortage in the urban water supply system. Finally, the calculation time increases with increasing problem dimensions, which is another proof of the model's validity (*Fig. 4*).

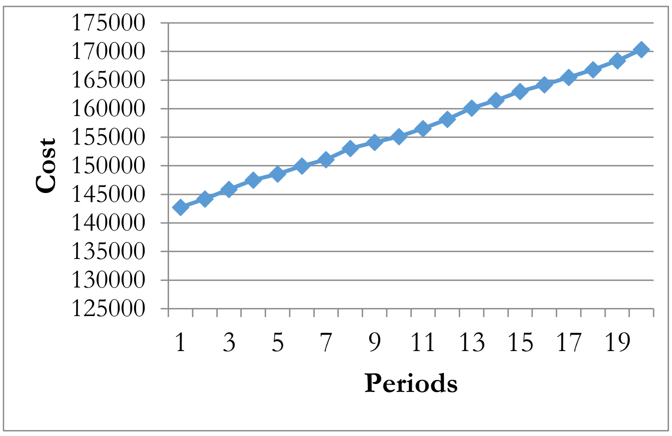

Fig. 2. The amounts resulting from the cost in different dimensions.

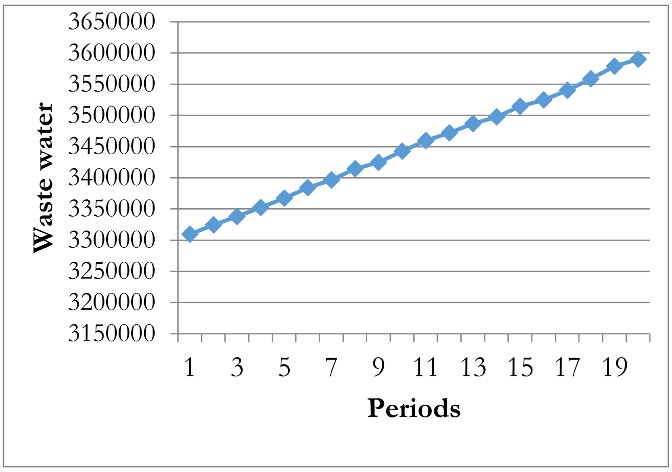

Fig. 3. The amounts of wastewater in different dimensions.

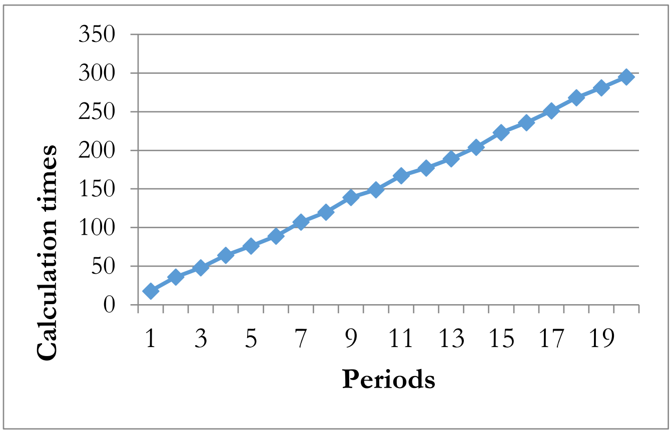

Fig. 4. The amounts obtained from the calculation time in different dimensions.

4.3 | Practical Results

Since the epsilon limit method has been introduced to solve the mathematical problem, the objective function values are calculated for different epsilon values and presented in *Table 5*. This model is solved using GAMS and CPLEX. *Table 5* presents different values of epsilon, and the objective functions are solved with these values. The values of the objective function show no significant change when increasing epsilon to a certain level. However, an increase in epsilon shows a considerable slope in the objective function values from some point onward, e.g., the first objective function. Such changes have indicated different slopes in the objective functions. Based on the obtained results, the feasible region and the improving vector of the objective functions were created for testing different values of epsilon. Accordingly, the significant changes in epsilon within the 50-900 interval were identified as the improving vector. Determining this interval indicates that the solution to the problem is outside the feasible region if epsilon is less than 50 or greater than 900. Thus, the epsilon value for searching the local optimum of the first objective function is set to 500, since the optimal solution occurs there. The optimal situation for the second objective function occurs at epsilon = 150, and the optimal solution range is 150-500 epsilon. *Table 5* presents the results of solving the model with a step length of 50 for epsilon.

Table 5. The results of solving the model using the epsilon constraint method.

Epsilon	Cost (Thousand)	The Amount of Wastewater and Shortage (Cubic Liters)	Final Execution Time (Minutes)
50	512	0.71	57
100	563	0.74	55
150	382	0.70	62
200	450	0.75	57
250	430	0.78	68
300	413	0.75	75
350	398	0.75	58
400	365	0.75	94
450	348	0.74	86
500	307	0.75	58
550	480	0.71	62
600	512	0.76	48
650	563	0.76	72
700	450	0.82	98
750	460	0.73	78
800	413	0.83	81
850	510	0.75	82
900	398	0.86	70
Optimum	307	0.70	48

4.4 | Sensitivity Analysis

This section provides a sensitivity analysis of the problem's significant parameters and compares the effectiveness of each parameter on the objective function values for the cost of water supply, the amount of wastewater, and the shortage. *Tables 6* and *7* present the results of this sensitivity analysis.

Table 6. Comparison of parameters affecting the urban water supply cost.

Water Supply Cost	Demand	Construction Cost	Resource Cost	Treatment Cost	Input Water	Capacity	Energy Consumption Cost	Energy Consumption Value	Transmission Cost
10%	0.008638	0.009408	0.011804	0.008477	-0.01212	-0.01212	0.011153	0.011258	0.008862
20%	0.019802	0.018301	0.023576	0.019319	-0.02285	-0.024	0.020425	0.021503	0.018325
30%	0.027447	0.031133	0.03255	0.032405	-0.03531	-0.03794	0.032352	0.0283	0.02519
40%	0.035404	0.041377	0.038717	0.042669	-0.0501	-0.05381	0.040323	0.037539	0.032173
50%	0.044155	0.050631	0.046318	0.048831	-0.06137	-0.06633	0.051125	0.04726	0.039032

As shown in *Table 6,* the parameters of Input water and capacity have the most significant negative effect on the change in water supply cost percentage. Thus, it worsens the water management solution. While other parameters have a positive effect and improve the solution. The effectiveness of the capacity and input water is 6.6% and 6.1%, respectively. In other words, increasing capacity can reduce costs by 7% while increasing the amount of input water by 6.1%, leading to a reduction and improvement in the solution. The capacity and the amount of input water are the only decreasing parameters, and the others have an increasing effect.

Table 7. Comparison of the parameters affecting the wastewater and shortage.

Water Supply Cost	Demand	Construction Cost	Resource Cost	Treatment Cost	Input Water	Capacity	Energy Consumption Cost	Energy Consumption Value	Transmission Cost
10%	0.004954	0.003617	0.003372	0.00512	-0.00359	-0.00323	0.003726	0.003234	0.005885
20%	0.009562	0.006929	0.008745	0.010833	-0.00904	-0.00762	0.008989	0.007001	0.008943
30%	0.014761	0.010269	0.013466	0.015987	-0.01303	-0.01349	0.013736	0.01253	0.011872
40%	0.019297	0.015422	0.017261	0.020491	-0.01802	-0.0184	0.01676	0.016575	0.015553
50%	0.024502	0.018362	0.02031	0.02548	-0.02342	-0.02295	0.021526	0.019683	0.019704

As shown in *Table 7,* the amount of input water and capacity reduces shortages and wastewater. At the same time, treatment cost and demand have the most significant effect on the shortage, at 2.5% and 2.4%, respectively. It should be noted that reducing input water by 2.5% reduces the shortage by 2.5%, and reducing capacity by 2.2% reduces the shortage by 2.2%.

4.5 | System Dynamics

This section evaluates the long-term performance of the urban water supply system dynamics model. Predetermined parameters are required to run the model, whose values are determined by the mathematical model. Thus, *Table 8* indicates the value of the mathematical model variables used in the system dynamics model.

Table 8. Determined parameters of the system dynamics model based on the mathematical model.

Parameter	Water Supply Resource		
	20	20	30
Y_{ik}	1555	1742	1853
Z_{jt}	**Treatment System**		
	12	14	16
	1222	1456	1525
WW_{it}	**Time**		
	10	12	15
	333	286	328
LW_{it}	**Time**		
	10	12	15
	1200	1500	1600

A cause-and-effect diagram is created according to *Fig. 5*, based on the values in *Table 8* and the solution of the mathematical model in the system dynamics model.

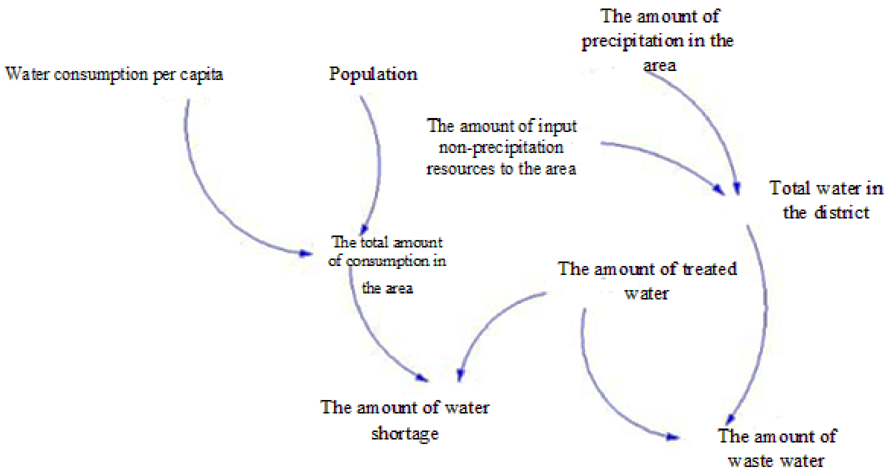

Fig. 5. Cause and effect diagram of the present study.

Fig. 6 shows that the per capita consumption rate is multiplied by the area's population, and the resulting value is the consumption. The amount of input water, along with precipitation, constitutes the total amount of water in the area. Finally, the amount of shortage is determined based on the amount of treated water and the amount of waste. To clarify the results, the flow storage diagram is presented based on *Fig. 6*.

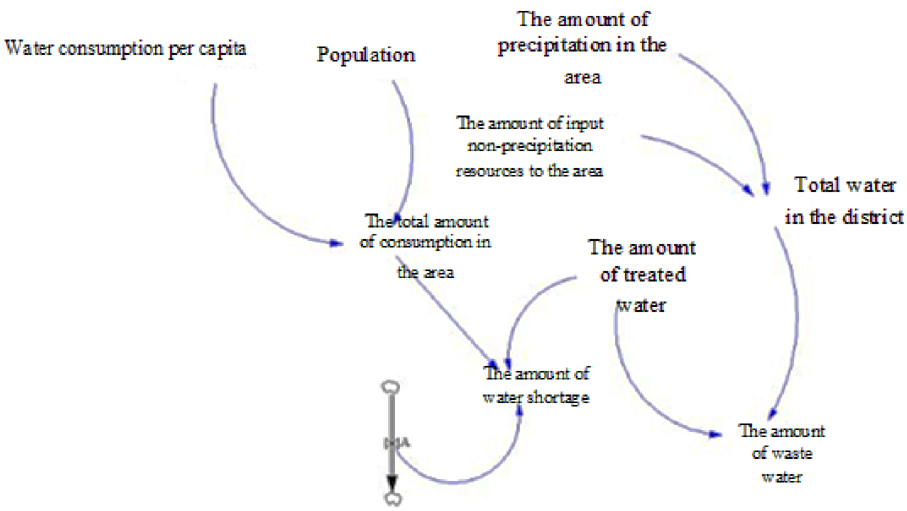

Fig. 6. Current flow storage diagram of the present study.

As shown in *Fig. 7*, the cause-and-effect relationships are presented in a quantitative diagram used to forecast the level of shortage. *Fig. 7* shows the final results of the shortage for the next 100 years in Guilan province.

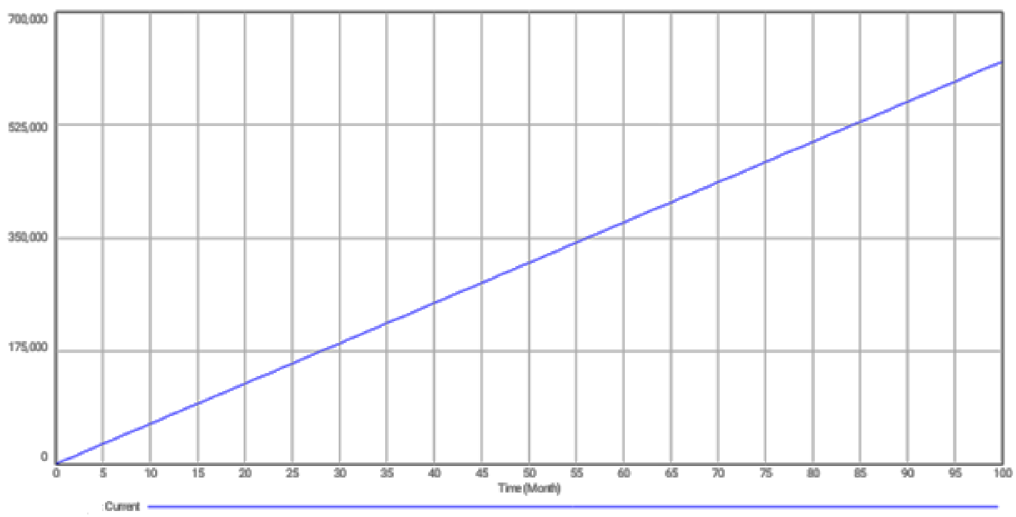

Fig. 7. Forecasting the amount of shortage.

Fig. 7 shows that the amount of shortage in the studied area will follow a linear trend during the next 100 years. In each period, the shortage increases with population growth and decreasing precipitation in the area. However, this increase does not follow an exponential distribution, and the amount of shortage can be expected in the studied area, given increases in population and precipitation.

5 | Conclusion

The present study aimed to optimize and plan the urban water supply system using bi-objective optimization and system dynamics method, and the case study was the Water and Wastewater Organization in Guilan province. For this purpose, library studies were conducted, and research gaps were identified based on these studies. The proposed method in this study was to provide a hybrid model of mathematical modeling, meta-heuristic algorithms, and system dynamics approach. The decision variables, including the amount of water resources in the reservoir, the amount of treated water in the water treatment system, the amount of wastewater in the area, and the amount of water shortage in the area, along with the amounts of the objective functions, were obtained using the mathematical model and meta-heuristic algorithm. Then, the amounts were entered into the system dynamics model as input values, and the shortage in future periods was predicted, indicating a linear trend of up to more than 520 thousand units. In addition, the sensitivity analysis shows the reverse effects of capacity and the amount of input water, and the direct effects of transmission cost, energy consumption, energy consumption cost, treatment cost, resource cost, construction cost, and demand. Among the parameters with a direct effect on energy consumption cost, construction cost, and treatment cost, construction cost has the most significant effect on supply cost. In contrast, treatment costs and demand have the most tremendous impact on shortages and wastewater in the system.

Based on the findings, the most significant suggestions for managers as a roadmap for the future are as follows: increasing treatment capacity and water resources in Guilan province can reduce treatment costs and alleviate shortages and wastewater. Thus, increasing investment to enhance treatment capacity and water resources is one of the approaches to prevent shortages and control wastewater, given the abundant water resources in Guilan province. Although the investment can be expensive in the short term, it prevents increases in costs and emerging water shortages in the long term. Regarding parameters that have a positive effect, such as the amount of non-precipitation water entering Guilan province from other provinces, appropriate measures should be taken through various methods to increase the province's water resources. In this regard, using input water can offer more advantages than using rainwater. If the right systems are used and managed effectively to control water inputs, an increase in water resources can be expected, resulting in reduced costs and wastewater in Guilan province. Energy consumption control is one of the factors that strongly affects the cost. Using devices with lower energy consumption can help treat and maintain water sources. Construction costs are considered an essential cost factor. Using existing resources appropriately and constructing new facilities can significantly affect costs. Demand is considered a significant factor in creating a shortage in the present model. The effect of demand on cost is not as significant as that of other parameters, though it can definitely affect shortages and wastewater. In this regard, acculturation regarding consumption and optimized consumption through the use of construction tools and valves can increase demand. Furthermore, presenting and analyzing the present model in the low-water provinces of Iran such as Yazd and Kerman, using more decision variables and parameters in the mathematical model, applying multiple meta-heuristic algorithms and comparing their performance, considering the uncertainty in some parameters such as the demand for wastewater, and using the system dynamics tool to predict the situation in the long term are considered as the most critical suggestions for the development of the present study in future studies.

Acknowledgments

The authors would like to express their sincere appreciation to all individuals whose valuable comments and support contributed to the successful completion of this study.

Funding

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Data Availability

The data supporting the findings of this study are available from the corresponding author upon reasonable request.

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